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Pedestrian modeling using the least action principle with sequences obtained from thermal cameras in a real life scenario

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***Abstract*: Living labs provide the possibility of doing real-time research in an ecological context corresponding to normal daily activities. In particular, it is important to know how humans respond to environmental changes and different scenarios. The appropriate characterization of individual human displacement dynamics within a crowd remains elusive, and for this reason, there is a keen interest in exploring behaviors with general physical models.**

In this work, we present a theoretical and experimental study of the natural movement of pedestrians when passing through a limited and known area of a shopping center. The modeling problem for the motion of a single pedestrian is complex and extensive; therefore we focus on the need to design models taking into account mechanistic aspects of human locomotion. The theoretical study used mean values of pedestrian characteristics, e.g., density, velocity, and many obstacles.

We propose a human pedestrian trajectory model by using the least-action principle, and we compared it against experimental results. The experimental study is conducted in a Living Lab inside a shopping center using infrared cameras. For this experiment, we collected highly accurate trajectories allowing us to quantify pedestrian crowd dynamics. The tests included 20 runs distributed over five days with up to 25 test persons.

Additionally, to gain a better understanding of subjects' trajectories, we simulated a background of different pathway scenarios and compared it with real trajectories. Our theoretical framework takes the minimum error between previously simulated and real point pathways to predict future points on the subject trajectory.

Keywords: infrared cameras, crowd dynamics, least-action principle, pedestrian simulation.

I. Introduction

Public areas designated for pedestrian traffic, such as underground corridors, shopping malls, airports among others, are places where multidisciplinary analysis of the interaction and behavior of people is of keen interest.

The study of pedestrian flow is not recent; it was developed for helping with the design and planning of buildings. It is also useful in the process of planning evacuations in emergency conditions. To improve service and conditions in pedestrian areas, it is necessary first to understand pedestrian interactions within the facility.

This paper explores paths of 25 pedestrians along a known area. After obtaining the trajectory and their points of origin, we evaluated the speed with the objective to calculate the kinetic force of the pedestrian. In the present model, we assume that the principle of least action holds and using this concept we can obtain the potential force. Once all the forces of pedestrian movement are known, then we calculate the adjustment of the parameters employed in the equations of the social force model.

The vector velocity, according to Weidmann [1], is different when taking into account three different characteristics. Pedestrian characteristics, such as age, weight, and sex. Trip characteristics (the purpose of the walk, familiarity of the route, the length of the path) and the structure properties (floor shape and size of the area).

If we consider all pedestrians to have the same psychological and physical characteristics as a case, the theory of unidimensional flow predicts a homogeneous distribution over the entire walking area with constant velocity [2].

When the flow is heterogeneous (pedestrians with different speeds), by changing the speed and direction of the pedestrian interactions, then pedestrian flow is distributed with various flux densities within the pedestrian area [3].

Presently, several models can be applied to pedestrian walk simulation. These models were intended to reproduce the pedestrian walking in a known area. Current models describing the trajectory of a pedestrian walking integrate the ideas of pedestrian behavior [4] and social behavior [5]. In this way, it is assumed that the trajectory can be induced for the so called "Social Force".

II. Social Force Model

The social force model [6] describes the interaction of the pedestrians with their environment (walls, obstacles, and another pedestrian etc.). The social force model is continuous in space and time. The equation of movement for every pedestrian uses Newton's second Law and is given as:

$$m_i \frac{dv_i}{dt} = F_i^{mov} + \xi_i. \quad (1)$$

Where ($i = 1, 2, \ldots$) indicates the pedestrian i; v_i shows the velocity of the pedestrian i; ξ_i term represents the alteration of the behavior, F^{mov} defines the pedestrian movement force which contains the sum of the forces, exerted on it, and is defined by:

$$F_i^{mov} = F_i^{kin} + F_{ij}^{rep} + F_{iw}^{pot} + F_{io}^{act}. \quad (2)$$

The Kinetic Force of the pedestrian i is calculated as follows: F_i^{Kin} is the force for the pedestrian with a mass m_i that is moving in $\vec{e}_i$ a direction with $\vec{v}_i$ velocity at a τ time.

$$F_i^{kin} = m_i \frac{\vec{v}_i^{\,0} - \vec{v}_i}{\tau}. \quad (3)$$

The Repulsion Force. Other pedestrians influence the pedestrian movement. Because of this, the pedestrian i will react by keeping a distance of value $\vec{R}_{ij}$ from the pedestrian j. It is observed that the closer the other person the greater the discomfort in the direction given by $\vec{e}_{ij}$. Thus, the Repulsion Force exerted by the pedestrian i is represented as:

$$F_{ij}^{rep} = -m_i \frac{v_{ij}^{\;2}}{\vec{R}_{ij}} \vec{e}_{ij}. \quad (4)$$

The Potential Force is the force exerted on the pedestrian due to the scenario. This force is what keeps pedestrians away from the edges of the stage, such as walls, and pillars. A least potential gradient represents the force; the vector $\vec{d}_{iw}$ is the vector distance between the pedestrian i and the wall or obstacle w closest to the individual.

$$\vec{F}_{iw}^{\;pot} = m_i \frac{A_w}{B_w} e^{-\frac{\|diw\|}{Bw}} \left(\frac{d_{iw}}{\|d_{iw}\|} \right). \quad (5)$$

The repulsive potential is monotonously decreasing [7]. Where A_w is the interaction between the edge and the pedestrian i, and B_w is the range of the interaction distance.

The Attraction Force is the force felt by the pedestrian i given the place or object g where he/she has an interest (Friends, arts, windows, products, and dressers) However, this force is usually determined by the interaction time. So these incentive effects can be modeled by a monotone decreasing potential [8] represented by:

$$f_{ig}\left(\|d_{ag}\|\right) = -\nabla_{d_{ig}} w_{ag}\left(\|d_{ig}\|\right). \quad (6)$$

The effects of attraction are responsible for making clusters of pedestrians. The final attractive force is given by:

$$F_{ig}(t) = m_i W\left(q_{ig}\right) f_{ig}\left(\|d_{ig}\|\right), \quad (7)$$

where $w(\theta_{ij})$ is the weight function, which depends on the angle of view of the pedestrian i.

The model equations describe the "social force" pedestrian behavior, but if we want to simulate all the forces that interact with each pedestrian, then the computational cost is high. For this reason, there is considerable interest in modeling the trajectory with simple models that have a good approximation for different scenarios.

III. Least Action Principle

From Newtonian mechanics, the principle of the least action describes the trajectory of a particle when considering the initial particle position $\vec{r}(t_i)$ on which acts a force $\vec{F}$ which may be the result of many forces as:

$$\vec{F} = m\vec{a} = m\ddot{\vec{r}}. \quad (8)$$

The objective of the model of classical mechanics is to solve the differential equation for different forces: gravity, and kinetic. For conservative forces, the force may be expressed concerning the potential.

$$\vec{F} = -\vec{\nabla} V, \quad (9)$$

Where the potential $\vec{\nabla} V$ depends of $\vec{r}$, but not or $\dot{\vec{r}}$. Newton equation is:

$$m\ddot{\vec{r}} = -\vec{\nabla} V. \quad (10)$$

This is a second order differential equation. The solution has two constants of integration; physically this means that we know the position and initial velocity to determine the final position.

For a particle that moves from an initial position $\vec{r}\left(t_1\right)$ to a final position $\vec{r}\left(t_2\right)$, without specifying the initial velocity, there exist n possible paths that connect these points. To find out which path the particle takes, we can take any route $\vec{r}\left(t\right)$ and call the defined action [9] as:

$$S\left[\vec{r}\left(t\right)\right]=\int_{t_1}^{t_2} dt\left(\frac{1}{2}m\dot{\vec{r}}^{\,2}-V\left(\vec{r}\right)\right). \quad (11)$$

The action is the difference between the kinetic energy (K.E) and potential energy (P.E) . The theory predicts that the path chosen by the particle is an extreme value of the action. The difference in the action for two different routes is:

$$\delta S = S\left[\vec{r}+\delta\vec{r}\right]-S\left[\vec{r}\right]= \int_{t_1}^{t_2}\left[-m\ddot{\vec{r}}-\vec{\nabla}V\right]\delta\vec{r}+\left[m\dot{\vec{r}}\cdot\delta\vec{r}\right]_{t_1}^{t_2}. \quad (12)$$

In the above equation if we evaluate at the edges when $\delta\vec{r}\left(t_1\right)=\delta\vec{r}\left(t_2\right)=0$. Then we have:

$$\delta S=\int_{t_1}^{t_2}\left[-m\ddot{\vec{r}}-\vec{\nabla}V\right]\cdot\delta\vec{r}. \quad (13)$$

The start condition of the path, when the action is zero is $dS=0$, this applies to all changes $\delta\vec{r}\left(t\right)$ that are made along the route. The only way this can be true is that the expression eq.(13) is zero. This is $m\ddot{\vec{r}}=-\vec{\nabla}V$ to require that the action is the extreme value is equivalent to follow the actual path. We can observe that the equation corresponds to Newton's equation.

The Lagrangian is the difference between the kinetic and potential energy. The total energy is the sum of these two energies. The theory predicts that the path chosen by the particle is an extreme value of the action that is a difference with the actual path must be zero.

The trajectory without obstacles for a pedestrian from an initial position to the final position can be described by the Lagrangian path equation.

In this case, the pedestrian moves freely to their destination without receiving external forces to change their trajectory i.e. its potential energy is zero, and the Lagrangian depends only on the kinetic energy.

$$L\left(\dot{\vec{r}},t\right)=\frac{1}{2}m\dot{\vec{r}}^{\,2}. \quad (14)$$

IV. Method

The Living Lab in Alexis Nihon shopping center in Montreal Figure 1(a) has provided the possibility of placing users in real-time ecological scenarios. This experiment is of great interest because it brings the possibility of including real user behaviors in the context of a general physical model.

For the experiments, we collect highly accurate trajectories that allow us to study the dynamics of pedestrian crowds. The experiments included 20 runs distributed over five days with up to 25 test persons.

Every run was filmed with at least two infrared cameras, which have been mounted at an altitude of 335 cm perpendicular to the floor. The assemblies can be seen in Figure 1(b). For the video analysis, we used MATLAB (R2012b). Pedestrian trajectories were recorded from different videos filmed on different days and hours to avoid any specific trend.

Figure 1. (a) Living Lab and **(b)** assemblies infrared cameras.

The data collection consists of the task associated with the observation and recording of pedestrian movement data, while the pedestrian analysis is focused on the interpretation of the data to understand the observed situation and to plan and design improvements.

This collection of data is an important step to ensure quality results. As in the case of crowding flow, pedestrian modeling also requires a high level of detail in data collection to be useful, including basic parameters like time, speed, and density.

The subjects are labeled looking for the center of the image of his face to facilitate a precise detection of his movement for the cameras. The three-dimensional (position and time) reconstruction of the face position was made from the digital movies of all three cameras Figure 2, encoded at 12 frames for second, and with the help of software developed in Mat Lab.

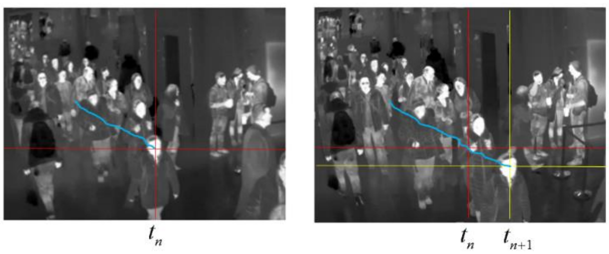

Figure 2. Pedestrian position in t_n and the next frame he/she is in next position t_{n+1}, the blue line is the virtual trajectory.

The pedestrian movement data for the analysis is collected using graphic video technique. A longitudinal trap of 15m to 20m is virtual mode made for measurement of velocity and trajectory.

The three-dimensional data were finally projected to the two-dimensional floor, and each pedestrian was characterized by a single point located in the middle of the line connecting both face positions Figure 3. The trajectories were finally smoothed over a time window of 10 frames.

To tackle this question, we have observed the behavior of a pedestrian moving in a corridor of the Living Lab under two experimental conditions:

(i) in the absence of interactions, and (ii) in response to a pedestrian moving in the opposite direction.

The comparison of pedestrian trajectories with and without reciprocal action allowed us to quantify the behavioral impact of the mutual action. The laws describing the influences were expressed mathematically and used in the social force model.

We then compared the predictions of the model with the experimental results and empirical data of pedestrian flows recorded in the living lab.
Finally, to obtain our final objective, the following methodology is utilized:

1. The size of the walking area (length and width of footpath).
2. Location of proper infrastructure (electric poles, and trees), and preferred walking areas.
3. Pedestrian detection.
4. Estimated trajectory
5. Data analysis.
6. Velocity and trajectory are used as the parameter for evaluation of simulation results in social force model.

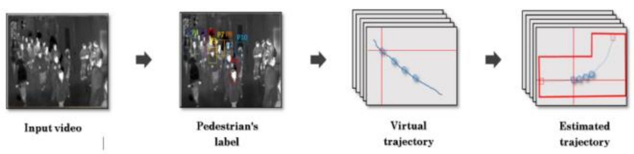

Figure 3. Methodology diagrams.

V. Data Collection

A) Determining Speed of Pedestrians

Pedestrian speed is the average pedestrian walking speed, generally expressed in units of meters per second. A virtual longitudinal trap, the length (12.7m) and width (8.3m), which is already known is marked on the ground for the observation of data through videos. The following are the steps involved in extracting the speed data:

1. Select a random pedestrian about to enter the trap and track him/her through the trap length.

2. Note the pedestrian's entry and exit times from the trap area, see Figure 3.

3. Pedestrian walking time is thus obtained by subtracting the time of entry into the rectangular box from the time of exit.

4. Walking speed is then derived by dividing the known length of the box by the walking time, previously calculated.

5. Registering of the trajectory.

From Figure 3, the total time taken by a randomly chosen pedestrian to cross the trap length of 12.7m is 9.4 seconds. Thus the speed of this random pedestrian is recorded as 1.34 m/sec. The average speed of random pedestrians taken at different intervals of time is recorded as the pedestrian speed at that particular location. At least one-third of the total pedestrians were sampled at each location to obtain the average pedestrian speed.

B) Determining pedestrian surface area density

Pedestrian density is the average number of pedestrians per unit of area within a walkway or queuing area, expressed as pedestrians per square meter. The following are the steps involved in extracting the pedestrian density data:

1. Rewind the tape back to the moment the test subject was located in the middle of the rectangular box.

2. As the selected pedestrian is in the middle of the study sur-face area, pause the tape and count the total number of other pedestrians in the rectangular box with the selected subject.

3. Dividing the total number of pedestrians in the box obtained from the previous step by the surface area of the rectangular box gives the pedestrian density.

Pedestrian surface area density is obtained by counting the total number of pedestrians in the pedestrian trap (Figure 4) and dividing it by the area of the pedestrian trap. Here, as shown in figure 4, the total number of pedestrians is 20. The area of the trap length is 105.4 m2. Thus the density of the study area was found to be 0.19 pedestrians for square meter. The average of densities recorded during the peak hour is reported as the density of the study area.

VI. Tracking Algorithm

For an optimal approximation of the kinetic and potential energy scenario, we obtained 100 videos of pedestrians walking in the study area. From those data, we selected ten pedestrians with a free path and ten pedestrians confronted to a pedestrian counter flow.

The first step was to identify and label every pedestrian see Figure 4(a), for every person we localized the centroid and manually registered the pedestrian position for every frame in the video. This was done to avoid any error from the tracking algorithm. With this new pedestrian position data, we used a method to correct the lens distortion of the camera and find the actual position on the scenario Figure 4(b).

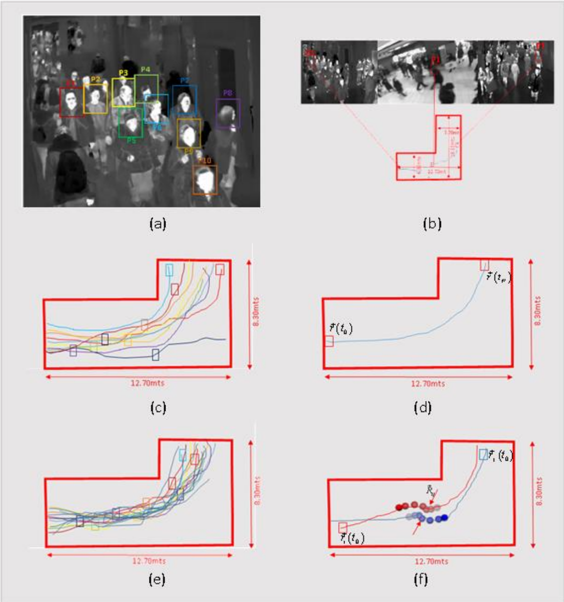

Figure 4. (a) This is the frame t = 0, from a video in which we labeled pedestrians to know their pathways from frame to frame till t = N. **(b)** We show the trajectory adjustment of the pedestrian starting position. The image is from two cameras we used. The area in red provides the surface where the study has been conducted. **(c)**We show ten pedestrian pathways within the scenario. **(d)** Blue is the averaged trajectory; x0 is the subject's path starting point, and xf is the end of the subject's path. **(e)** We show 20 trajectories and ten interactions between pedestrians. **(f)** Finally, in this image, we show the interaction of two pedestrians. After the trajectories have been obtained, we registered two different velocity groups: the velocity of free walking and the velocity of the pedestrians meeting an obstacle.

The principle of least action implies that the value of the kinetic force is minimized. That is, theoretically, the pedestrian takes the path that represents less effort. In the absence of a potential, the trajectory is perfectly described by the action from the average speed of the pedestrian, which is calculated using the ratio of the total distance over the total time. In this way, the pedestrian can walk faster at the beginning and then reduce its speed. However, to minimize the action, the pedestrian must go at a uniform speed.

In real cases, the pedestrian modifies its velocity along the trajectory, because of the Potential Energy of the environment and the interaction with the obstacles, as predicted by the Social Force Model. In every case, the action is equal to zero when the trajectory is the real one.

Let's consider that for the actual trajectory of the pedestrian at a time t, from an initial point $\vec{r}_1 = \vec{r}\left(t_0\right)$ to a final point $\vec{r}_2 = \vec{r}\left(t_N\right)$ it is assumed that the action is equal to zero. Knowing the mass of the pedestrian $\left(m_i\right)$ you can then obtain the value of the kinetic energy, in theory, it is possible to recover the value of the potential energy for each scenario and every Dt of the trajectory.

The next step is to find the values of all the potential force parameters involved to help us gauge the scenario under study and have a better social force model amending pedestrian path. As described before, there are n interaction cases, the main cases for this work are free walking, repulsive force towards an obstacle and attraction force. We will discuss only the first two, and we propose the basis for the experiment in the Living Lab for attractive forces.

VII. Results

To demonstrate that our theoretical model can get the value of the potential energy that makes the action equal to zero. We took ten trajectories from different pedestrians (See Method). We also calculated the average of all free walking trajectories for every position and obtained the average trajectory shown in Figure 4(d). We assumed the mass of every pedestrian as $\bar{m} = 80kg$.

Description		**Average result**
Average velocity	$\bar{v}$	1.34m/s
Average kinetic force	$\vec{F}_f^{kin}$	54.76 N
Average potential force	∇V	0 N
Range of the interaction distance walls	$\vec{B}_w$	0.8m
Interaction force parameter of pedestrian-walls	$\vec{A}_w$	9. 4m^2/s^2

Table 1. Free walking results.

For trajectory with an obstacle results we obtained the next valued:

Description		**Average result**
Average velocity	$\bar{v}$	1.17m/s
Average distance vector between two pedestrian	$\left\|\vec{R}_{ij}\right\|$	0.48m
Average kinetic force	$\vec{F}_i^{kin}$	62.03N
Magnitude average social discomfort	$\left\|\vec{e}_{ij}\right\|$	0.2m
Average repulsion force	$\left\|\vec{F}_i^{rep}\right\|$	-8.003N
Interaction force parameter of pedestrian-walls	$\vec{A}_w$	3.4m^2/s^2

Table 2. Trajectory with an obstacle results.

VIII. Simulation of results

In this section, we present the numerical simulations of distinct stages where the pedestrians interact. The model used is "social force" formally defined in the equations of an earlier section.

Movement is carried out in two parts. First, the complete path of every pedestrian is calculated, and later his positions are updated to the new cell. Every passage of time represents one cut in sub passages of time. The algorithm of movement is applied to every pedestrian in every sub-step of time to complete the whole passage of time or the single speed. The update of the positions of the pedestrians is done in parallel, that is to say, with every passage of time, the pedestrians move to the last cell of the calculated path.

Calculation of pedestrian paths involves the following steps:
Step 1. Calculate the field of vision of the pedestrian.
Step 2. Calculate socially weighted values of pedestrian's line of trek.
Step 3. If the socially weighted value is less then one, then must check if the cell is unoccupied and go to step six. If not go to the step four.
Step 4. Calculate the value of every socially weighted parameter for the adjacent lines of trek.
Step 5. Compare and choose the vacated cell with minor social weighted values.
Step 6. Move and update the social field of the pedestrian, go to step seven.
Step 7. In the case of lack of a vacated cell then the pedestrian remains in the same cell.
Step 8. End

The simulation was performed using the Social Force model described in section 2 with parameters obtained in our analysis. The simulation is performed separately for each scenario described above. Although a shoulder width of 45.58 cm is proposed [10](Still 2000), we considered in this example 80 cm wide shoulders that gave better results. We obtained the average speed $\bar{v} = 1.34\frac{m}{s}$ with a standard deviation of 0.26m/s. For simplicity, the attractive forces or fluctuations $\xi(t)$ are not considered. For this situation, we have examined the parameters in Table 1. We used the Euler method for calculating the velocity of every pedestrian at the time $t + \Delta t$ as follows:

$$v_i(t+\Delta t) = v_i(t) + F_i^T(t)\Delta t, \quad (15)$$

Where Δt is the time interval. After, we calculated the position $\vec{r}_i$ of every pedestrian:

$$\vec{r}_i(t+\Delta t) = \vec{r}_i(t) + \vec{r}_i(t+\Delta t). \quad (16)$$

For the simulation of the repulsion force between pedestrians, we considered two of them walking one towards the other as can be seen in Figure 6. We used eq. 4 considering the pedestrian as the center of a radial force, specification circular [11]. The parameters in Table 2 are obtained using the principle of least action. The hall measures are 12.7 meters in length and 8.3 meters in width. The area has a constant radius of 0.7 meters. The first real distance between subjects is 4 meters. When they are close enough to a certain distance determined by $\vec{R}_{ij}$ both pedestrians begin to feel the repulsion force between each other, modifying both trajectories.

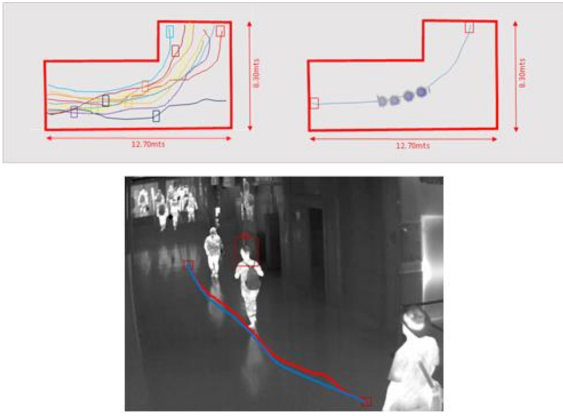

Figure 5. (Left) Ten trajectories without obstacles for ten pedestrians. (Right) trajectories' averages in the scenario. In this case, the value for the potential energy represents the mean that the scenario exerts on the pedestrian. (Bottom) In blue we show the real trajectory of a free walking pedestrian. In red we show the trajectory that was obtained from the social force model applying the parameters that we obtained with our analysis.

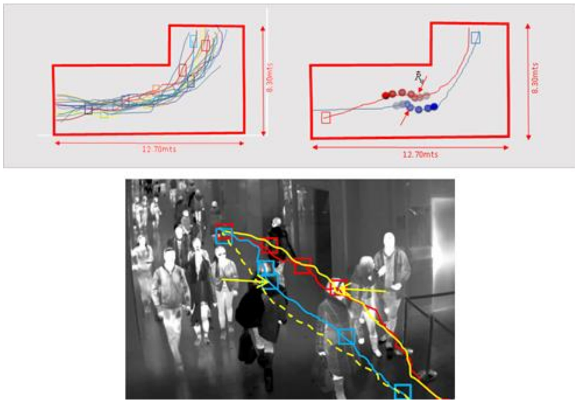

Figure 6. (Left) Ten trajectories with obstacles of ten pedestrians. (Right) modified path of a pedestrian that interacts with another pedestrian. (Bottom) blue and red lines show trajectories of the real interaction of two pedestrians walking counter flow. In yellow (solid and broken) we show the simulated trajectories.

To evaluate the validity of our estimations, two types of error index are used in this study, a mean-absolute-relative error (MARE) and a root-mean-square error (RMSE). The formal expression of MARE and RMSE are presented in the following equations.

$$MARE = \frac{1}{N}\sum_{ij=1}^{N}\frac{\left|PT_{ij} - \hat{PT}_{ij}\right|}{PT_{ij}},$$
$$RMSE = \sqrt{\frac{1}{N}\sum_{ij=1}^{N}\left(PT_{ij} - \hat{PT}_{ij}\right)^2}. \quad (17)$$

Where PT_{ij} is the real position of the pedestrian and $P\tilde{T}_{ij}$ is the position calculated by the simulation. We obtained 25 trajectories of pedestrians. All the trajectories had similar interactions.

The MARE and RMSE are 7.21% and 3.96% in these cases. The errors are relatively small, and the results confirmed that the proposed model predicts the pedestrian trajectories.

IX. Conclusion

It is possible to reproduce observed results for real pedestrian movement by using the Least Action Principle. In the first scenario, we focused on a pedestrian walking without obstacles. Using the actual trajectories of the experiment we obtained the necessary information and applied it to the Social Force Model. Our simulations were clearly able to reproduce the actual observed average trajectories for the free obstacle walking conditions. When a scenario does not represent free walking (obstacles, constraints), the potential energy and the kinetic energy are modified. Note that when the trajectory is real, the action is assumed to equal zero. That is the value of the potential energy changes in each interaction with a new obstacle. However, the value of the action remains. It is shown here that we can clearly reproduce some scenarios and calibrate the model according to different situations. Using different values of potential energy, we can obtain the values of the actual pathway. Nevertheless, as a significant extension concerning this model, it would be desirable to simulate cellular automata that could learn the situation and improve the approximation model to predict the real trajectories with more accuracy. It should be acknowledged that the proposed suggestions are based on the assumption that pedestrians are equally distributed on each side of their pathways, which is a situation with extreme bi-directional effects. The purpose for this assumption is to make sure that the recommended pathway parameters satisfy the most demanding situations. For further refined evaluation, since our simulation model can be flexibly expanded, a variety of scenarios can also be evaluated by varying the arrival pattern of pedestrians at each side of their pathways. Furthermore, we should point out that the proposed methodology does not consider pedestrian trip purposes and conditions where, in our case, a significant proportion of pedestrians were older clients and young students. The next step is to apply different forces of repulsion and attraction during the pedestrian walk in the Living Lab, to modify its trajectory and ascertain whether this scenario simulation predicts real path.

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Eduardo Lugo, received his PhD degree in physics from Morelos State University, Mexico. He was a professor at Morelos State University and the National Autonomous University of Mexico (UNAM). He was a postdoctoral fellow at University of Rochester and McGill University. Currently, he is at Faubert Lab, working with different photonic applications ranging from photonic crystals and near-infrared spectroscopy of the brain.

Rafael Doti, is an electronic and mechanical engineer from the engineering faculty of Buenos Aires University (FIUBA), Argentina. He was a researcher at the FIUBA Laser Laboratory and he taught physics at the Physics Department (FIUBA). He joined Faubert Lab at Montreal University, where he has helped develop new methods, experiments, and devices for sensory stimulation and data record, Shack–Hartmann wave front analysis, and near-IR brain spectroscopy.

Jocelyn Faubert received his PhD degree in psychophysics from Concordia University. He then started a postdoctoral fellowship at Harvard University. He is full professor at University of Montreal at the School of Optometry. He is the director of the Visual Psychophysics and Perception Laboratory. He is a senior chair holder for the Natural Sciences and Engineering Research Council-Essilor Industrial Research Chair. He was founder of Ophtalox, Inc., CogniSens, Inc., and CogniSens Athletics, Inc. Companies.